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Published in:
Physical Review Letters

DOI:
10.1103/PhysRevLett.102.201301

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2009

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):
Bergshoeff, E. A., Hohm, O., & Townsend, P. K. (2009). Massive Gravity in Three Dimensions. *Physical Review Letters*, *102*(20), [201301]. https://doi.org/10.1103/PhysRevLett.102.201301

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Massive Gravity in Three Dimensions

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(Received 21 January 2009; published 21 May 2009)

A particular higher-derivative extension of the Einstein-Hilbert action in three spacetime dimensions is shown to be equivalent at the linearized level to the (unitary) Pauli-Fierz action for a massive spin-2 field. A more general model, which also includes "topologically-massive" gravity as a special case, propagates the two spin-2 helicity states with different masses. We discuss the extension to massive $\mathcal{N}$-extended supergravity, and we present a "cosmological" extension that admits an anti–de Sitter vacuum.

DOI: 10.1103/PhysRevLett.102.201301 PACS numbers: 04.60.Kz

For some purposes it is useful to think of Einstein's theory of gravity, general relativity (GR), as a model for the consistent interaction of a massless spin-2 field on a four-dimensional (4D) Minkowski "background." This perspective makes it clear that the quanta associated with gravitational waves, i.e., gravitons, are massless particles with two independent polarization states of helicity ± 2. Unfortunately, the quantum theory of gravitons is nonrenormalizable, a fact that has led many theorists to consider GR and its variants in three dimensions (3D) because one expects less severe short-distance behavior in a lower dimension. Pure GR is perhaps too simple for this purpose because its linearization on a Minkowski vacuum yields an equation that propagates no physical helicity states [1]. A popular modification of GR in 3D is the "topologically-massive gravity" (TMG), which complements the Einstein-Hilbert (EH) action with a Lorentz Chern-Simons (LCS) term [2], thus breaking parity as well as introducing a new mass scale. Linearization yields a third-order wave equation but, remarkably, the theory is unitary and propagates a *single* massive mode of helicity ± 2, the sign depending on the sign of the LCS term.

The main aim of this Letter is to present a different, *parity-preserving* variant of 3D GR that describes, on quantization, a unitary interacting theory of gravitons, each of which has *two* polarization states of helicity ± 2, as in 4D GR except that the 3D graviton is massive. This is so despite the fact that the field equations are fourth order in derivatives, because linearization of our "massive gravity" theory yields a free "fourth-order" theory with precisely the required physical content; we confirm this result by a simple proof of its equivalence to the (3D) Pauli-Fierz (PF) "second-order" theory for a massive spin-2 field. In 4D, this type of "higher-derivative" theory is not unitary but it *is* renormalizable [3], and this implies super-renormalizability in 3D.

The representation theory of the Poincaré group is essentially the same for massive 3D particles as it is for massless 4D particles. However, the CPT theorem in 4D implies that every state of helicity h is accompanied by a state of helicity $-h$, with the same mass. In contrast, the masses may differ in 3D, at the cost of violating parity. Furthermore, given a pure spin s theory, with helicities $\pm s$, we may take the mass of one helicity state to infinity, thereby arriving at a theory describing a single helicity s state. In the $s = 1$ case, this decoupling of 3D helicity states is reflected in the fact that the Proca equation factorizes into two first-order equations, each describing one helicity state [4]; the helicity content of this "square root" of the Proca theory is therefore identical to that of the topologically-massive spin-1 theory [5], to which it may be shown to be equivalent via a "master action" [6]. A similar factorization occurs for the 3D Pauli-Fierz equation for spin 2 [7], and the resulting first-order equation has been shown [8] to be equivalent to linearized TMG via an equivalence of both to an intermediate "self-dual" theory [9]. In other words, the linearized TMG field equations are equivalent to the square root of field equations that are themselves equivalent to the linearized equations of our new massive gravity theory. Here we unify the spin-2 equivalences that underlie this interpretation by means of a "triple-master" action.

We begin with a presentation of the new massive gravity theory. Let $g_{\mu\nu}$ ($\mu,\ \nu = 0,\ 1,\ 2$) be the 3D spacetime metric, with determinant g, and let $R_{\mu\nu}$ be its Ricci curvature tensor, which determines not only the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$ but also the full Riemann tensor. We choose the $(+--)$ metric signature. Now consider the action

$$S = \frac{1}{\kappa^2}\int d^3x\sqrt{g}\Big[R + \frac{1}{m^2}K\Big], \qquad (1)$$

where

$$K = R_{\mu\nu}R^{\mu\nu} - \tfrac{3}{8}R^2. \qquad (2)$$

The constant κ, which has mass dimension $[\kappa] = -1/2$ in fundamental units, is the 3D analog of the square root of

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Newton's constant, while m is a "relative" mass parameter, which could be traded for the effective dimensionless coupling constant $m\kappa^2$, as for TMG [10]. Also in common with that theory is the "wrong" sign for the EH term. Remarkably, this higher-derivative theory is unitary, with field quanta that are massive spin-2 particles, each with two polarization states, of helicity 2 and -2.

This result may be established in various ways. Let us begin by considering the field equations. These are

$$2m^2 G_{\mu\nu} + K_{\mu\nu} = 0, \tag{3}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the Einstein tensor, and

$$\begin{aligned} K_{\mu\nu} = {} & 2D^2 R_{\mu\nu} - \tfrac{1}{2}(D_\mu D_\nu R + g_{\mu\nu} D^2 R) - 8R^\rho_\mu R_{\nu\rho} \\ & + \tfrac{9}{2} R R_{\mu\nu} + g_{\mu\nu}[3R^{\rho\sigma} R_{\rho\sigma} - \tfrac{13}{8} R^2], \end{aligned} \tag{4}$$

where D_μ is the usual Levi-Civita covariant derivative, and $D^2 \equiv D^\mu D_\mu$. As a consequence of the diffeomorphism invariance of the action, we have the Bianchi-type identity $D^\mu K_{\mu\nu} \equiv 0$. A *special* feature of the scalar K, and the tensor $K_{\mu\nu}$ derived from it is that

$$g^{\mu\nu} K_{\mu\nu} = K. \tag{5}$$

As a consequence, the trace of (3) yields

$$m^2 R = K. \tag{6}$$

Note, in particular, the absence on the right-hand side of a $D^2 R$ term, which would contribute to the linearized equation if it were present.

The next step is to linearize the field equations about the Minkowski vacuum solution that they obviously admit by writing $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ for the Minkowski metric $\eta_{\mu\nu}$ and perturbation $h_{\mu\nu}$. The linearized field equations are then found to be

$$(\Box + m^2) G^{\rm lin}_{\mu\nu} = 0, \qquad R^{\rm lin} = 0, \tag{7}$$

where $G^{\rm lin}_{\mu\nu}$ is the linearized Einstein tensor, and $R^{\rm lin}$ the linearized Ricci scalar, which is zero by (6) since K contains no term linear in the metric perturbation. The linearized Einstein tensor may be written in the form $[\mathcal{G}h]_{\mu\nu}$, where $\mathcal{G}$ is the following linear differential operator, which we call the "Einstein" operator:

$$\mathcal{G}_{\mu\nu}{}^{\rho\sigma} = \tfrac{1}{2} \varepsilon_{(\mu}{}^{\eta\rho} \varepsilon_{\nu)}{}^{\tau\sigma} \partial_\eta \partial_\tau. \tag{8}$$

In momentum space, we may view $\mathcal{G}$ as a 6×6 matrix. This matrix is not invertible but it maps the three-dimensional "physical" subspace of transverse metric perturbations to itself, and so defines a projected 3×3 matrix $\mathcal{G}_\perp$. This projected matrix *is* invertible; it is for this reason that the 3D Einstein equation propagates no physical modes. The three eigenvectors of $\mathcal{G}_\perp$ are two traceless transverse metric perturbations describing massive modes of helicities ± 2, and the transverse trace, which describes a massive mode of zero helicity. However, $\mathcal{G}_\perp$ is an operator of no definite sign; the eigenvalues of the helicity ± 2 modes are negative whereas the eigenvalue of the zero helicity mode is positive. This implies (given our conventions) that the helicity ± 2 modes are physical whereas the zero helicity mode would be a "ghost" (i.e., have negative kinetic energy) were it not for the $R^{\rm lin} = 0$ constraint that removes precisely this mode.

An implication of the foregoing is that linearized new massive gravity is equivalent to the Pauli-Fierz theory for a free massive spin-2 field. We will now demonstrate this equivalence directly. We begin with the observation that (1) is equivalent to the action with Lagrangian density

$$\mathcal{L} = \frac{1}{\kappa^2} \sqrt{g} \left[R + f^{\mu\nu} G_{\mu\nu} - \frac{1}{4} m^2 (f^{\mu\nu} f_{\mu\nu} - f^2) \right], \tag{9}$$

where $f_{\mu\nu}$ is an auxiliary symmetric tensor field with trace $f = g^{\mu\nu} f_{\mu\nu}$. Next, we expand about a Minkowski background, keeping only quadratic terms in the metric perturbation. The result is

$$\mathcal{L}_2 = (f^{\mu\nu} - \tfrac{1}{2} h^{\mu\nu})[\mathcal{G}h]_{\mu\nu} - \tfrac{1}{4} m^2 (f_{\mu\nu} f^{\mu\nu} - f^2). \tag{10}$$

Naturally, elimination of $f_{\mu\nu}$ yields the quadratic approximation to (1) about the Minkowski background solution, but we may instead eliminate $h_{\mu\nu}$; its field equation is $\mathcal{G}(h - f) = 0$. Because the Einstein operator is invertible on the space of transverse symmetric tensors, the solution of this equation is $h_{\mu\nu} = f_{\mu\nu}$ up to a linearized gauge transformation which is irrelevant because the action is gauge invariant. Back substitution now yields an action with Lagrangian density

$$\mathcal{L} = \tfrac{1}{2} f^{\mu\nu} [\mathcal{G}f]_{\mu\nu} - \tfrac{1}{4} m^2 (f^{\mu\nu} f_{\mu\nu} - f^2). \tag{11}$$

This is precisely the Pauli-Fierz theory for a massive spin-2 field $f_{\mu\nu}$. The first term is the linearization of the EH term, which now has the "right" sign.

To further understand what is so special about the action (1), it is useful to consider it as a special case of the class of models in which K is replaced by $aK + bR^2$ for constants (a, b), not both zero. These models were investigated in [11] for a "right-sign" EH term, in which case there are tachyons unless $a \leq 0$ and $b \geq 0$, and only $a = 0$ yields a ghost-free model, which propagates a single scalar mode. Actually, it is well known, at least for 4D, that a Lagrangian density of the form $\mathcal{L} = \phi(R)$ is equivalent, for some "suitable" class of functions ϕ, to GR coupled to a scalar field with a potential determined by the function ϕ; the history is summarized in [12], where the extension to all $D \geq 3$ is also presented. For the "wrong-sign" EH term, there are tachyons unless $a \geq 0$ and $b \leq 0$, and only $b = 0$ yields a ghost-free model, which propagates spin-2 modes of helicity $+2$ and -2.

An analysis of the $\mathcal{N} = 1, 2$ supergravity extensions of the higher-derivative gravity with $K \to aK + bR^2$ was also presented in [11], again for the right-sign EH term but the detailed results can be used to deduce some interesting consequences for "massive supergravities" which we define to be the supersymmetric extensions for $(a, b) = (1, 0)$ with the wrong-sign EH term. It will suffice to consider the bosonic fields. For $\mathcal{N} = 1$ there is a scalar "auxiliary" field S which actually is auxiliary for $b = 0$. So the $\mathcal{N} = 1$ massive supergravity is ghost-free and propagates a supermultiplet of spins $(2, 3/2)$. For $\mathcal{N} = 2$ there is a complex scalar auxiliary field which is, again, actually auxiliary only for $b = 0$. There is also a real vector auxiliary field; remarkably, its action is the Proca action for $b = 0$. So the $\mathcal{N} = 2$ massive supergravity is ghost-free and propagates a supermultiplet of spins $(2, 3/2, 3/2, 1)$ (each with two helicities). It is fairly clear that an $\mathcal{N} = 4$ massive supergravity could be constructed similarly, using a "tensor calculus" derived from the $\mathcal{N} = 2$ tensor calculus in 4D, but beyond that we can only speculate.

We now aim to make contact with the parity-violating topologically-massive gravity. We start from a triple-master action that depends on three second-rank tensor fields (h, k, e) on 3D Minkowski spacetime. We assume that h is a symmetric tensor but that k and e are *general* second-rank tensors. The Lagrangian density is

$$\mathcal{L}(h,k,e) = -\frac{1}{2\mu^2}(\mu h + 2k)^{\mu\nu}[\mathcal{G}(\mu h + 2k)]_{\mu\nu} + \frac{1}{\mu}\varepsilon^{\mu\nu\rho}(e+k)_\mu{}^\sigma\partial_\nu k_{\rho\sigma} - \frac{1}{4}(e^{\nu\mu}e_{\mu\nu} - e^2), \tag{12}$$

where μ is a mass parameter. Elimination of the auxiliary field e yields

$$\mathcal{L}(h,k) = -\frac{1}{2\mu}(\mu h + 4k)_{\mu\nu}[\mathcal{G}h]^{\mu\nu} + \frac{1}{\mu}\varepsilon^{\mu\nu\rho}k_\mu{}^\alpha\partial_\nu k_{\rho\alpha}. \tag{13}$$

The k equation of motion has the solution

$$k_\mu{}^\nu = \tfrac{1}{2}\varepsilon^{\nu\sigma\lambda}\partial_\sigma h_{\lambda\mu} + \partial_\mu\xi^\nu, \tag{14}$$

for the arbitrary vector field ξ, which drops out on back substitution; we thus get the Lagrangian density of linearized TMG:

$$\mathcal{L}(h) = -\frac{1}{2}\left(h_{\mu\nu} + \frac{1}{\mu}\varepsilon_\nu{}^{\tau\sigma}\partial_\tau h_{\sigma\mu}\right)[\mathcal{G}h]^{\mu\nu}. \tag{15}$$

Note that the EH term has the expected wrong sign. Thus, the triple-master action is equivalent to linearized TMG, but we now obtain two other equivalent actions as follows.

Returning to (13), we see that the h equation of motion implies that $h_{\mu\nu} = -(2/\mu)k_{(\mu\nu)}$ modulo an irrelevant gauge transformation, and back substitution then yields the Lagrangian density

$$\mathcal{L}(k) = \frac{2}{\mu^2}k^{\mu\nu}[\mathcal{G}k]_{\mu\nu} + \frac{1}{\mu}\varepsilon^{\mu\nu\rho}k_\mu{}^\sigma\partial_\nu k_{\rho\sigma}. \tag{16}$$

Note that the linearized EH term for k has the right sign, and that the second term depends on both the symmetric and antisymmetric parts of $k_{\mu\nu}$. This is the self-dual model of [9].

Alternatively, we can return to (12) and eliminate h. Its equation of motion again implies that $h_{\mu\nu} = -(2/\mu)k_{(\mu\nu)}$ modulo an irrelevant gauge transformation, and back substitution then gives

$$\mathcal{L}(k,e) = \frac{1}{\mu}\varepsilon^{\mu\nu\rho}(k+e)_\mu{}^\sigma\partial_\nu k_{\rho\sigma} - \frac{1}{4}(e^{\nu\mu}e_{\mu\nu} - e^2). \tag{17}$$

The equation of motion for k is

$$\varepsilon^{\mu\tau\rho}\partial_\tau(2k_\rho{}^\nu + e_\rho{}^\nu) = 0, \tag{18}$$

which implies that $k_\mu{}^\nu = -\frac{1}{2}e_\mu{}^\nu + \partial_\mu\xi^\nu$ for arbitrary vector ξ which, as before, drops out upon back-substitution to leave us with the Lagrangian density

$$\mathcal{L}(e) = -\frac{1}{4\mu}\varepsilon^{\mu\nu\rho}e_\mu{}^\sigma\partial_\nu e_{\rho\sigma} - \frac{1}{4}(e^{\nu\mu}e_{\mu\nu} - e^2). \tag{19}$$

This is the "first-order" spin-2 model of [7]. Its equations of motion can be shown to be equivalent to

$$\varepsilon^{\mu\tau\lambda}\partial_\tau e_\lambda{}^\nu + \mu e^{\nu\mu} = 0, \qquad \eta^{\mu\nu}e_{\mu\nu} = 0, \qquad e_{[\mu\nu]} = 0. \tag{20}$$

Iteration of the first-order differential equation, which implies that $\partial_\mu e^{\nu\mu} = 0$, yields the Klein-Gordon equation for $e_{\mu\nu}$, which is equivalent to the PF equations when combined with the algebraic constraints. However, because the original equation was first order, only one of the two spin-2 modes of the PF theory is propagated.

The equations of motion of the triple-master action imply that $e_{\mu\nu} = (2/\mu)[R^{\rm lin}_{\mu\nu} - (1/4)\eta_{\mu\nu}R^{\rm lin}]$, and using this in (20) we arrive at the linearized TMG equations in the form

$$\mathcal{O}_\mu{}^\rho(\mu)G^{\rm lin}_{\rho\nu} = 0, \qquad R^{\rm lin} = 0, \tag{21}$$

where $\mathcal{O}$ is the operator of the self-dual spin-1 theory [4]:

$$\mathcal{O}_\mu{}^\nu(\mu) = \delta_\mu{}^\nu + \frac{1}{\mu}\varepsilon_\mu{}^{\tau\nu}\partial_\tau. \tag{22}$$

The tensor $\mathcal{O}G^{\rm lin}$ is symmetric, despite appearances, as a consequence of the linearized Bianchi identity. Let us now consider the alternative equations

$$[\mathcal{O}(-m_-)\mathcal{O}(m_+)]_\mu{}^\rho G^{\rm lin}_{\rho\nu} = 0, \qquad R^{\rm lin} = 0. \tag{23}$$

Evidently, these propagate helicities ± 2 with masses $m_\pm$, so we recover (21) by taking $m_- \to \infty$ for fixed $m_+ = \mu$. If instead we set $m_+ = m_- = m$ then we get the parity-preserving equations (7). In this sense, TMG is a "square root" of the new massive gravity proposed here, but both are actually special cases of a "general massive gravity" (GMG) theory with two mass parameters. To see this, we observe that the equations (23) are equivalent to the linearization of the equation

$$G_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \frac{1}{2m^2} K_{\mu\nu} = 0, \tag{24}$$

where $C_{\mu\nu}$ is the Cotton tensor,

$$C_{\mu\nu} = (1/\sqrt{g})\varepsilon_\mu{}^{\tau\rho} D_\tau [R_{\rho\nu} - (1/4) g_{\rho\nu} R], \tag{25}$$

which arises from variation of a LCS term, and

$$m^2 = m_+ m_-, \qquad \mu = m_+ m_- / (m_- - m_+). \tag{26}$$

For $m \to \infty$ for fixed μ we recover TMG while $\mu \to \infty$ for fixed m yields the model defined by (1).

We now turn to the cosmological extension of the GMG model obtained by adding a cosmological term to the field equation (24), as recently considered for TMG [13,14]. Specifically, we consider the field equation

$$\lambda m^2 g_{\mu\nu} + \alpha G_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} + \frac{\beta}{2m^2} K_{\mu\nu} = 0, \tag{27}$$

where λ is a dimensionless parameter, as are α and β, which we include for generality. Let us seek maximally symmetric vacuum solutions for which

$$G_{\mu\nu} = \Lambda g_{\mu\nu}, \tag{28}$$

for some "cosmological" constant Λ; for such solutions we have $C_{\mu\nu} = 0$ and $K_{\mu\nu} = -\frac{1}{2}\Lambda^2 g_{\mu\nu}$. If $\beta = 0$, but $\alpha \neq 0$, the field equation is solved when $\Lambda = -(\lambda/\alpha) m^2$, which is the anti–de Sitter (AdS) vacuum of "cosmological TMG" for $\lambda/\alpha > 0$. If $\beta \neq 0$ then the field equation is solved when

$$\beta\Lambda = 2m^2[\alpha \pm \sqrt{\alpha^2 + \beta\lambda}] \qquad (\beta \neq 0). \tag{29}$$

There is no maximally symmetric vacuum unless $\beta\lambda \geq -\alpha^2$, and when this inequality is saturated there is a unique vacuum that is de Sitter (dS) for $\beta/\alpha > 0$ and AdS for $\beta/\alpha < 0$. Given $\alpha \neq 0$, there are two inequivalent (A)dS vacua when $0 > \beta\lambda > -\alpha^2$, one of which becomes the Minkowski vacuum of our original massive gravity theory in the limit that $\lambda = 0$. For $\beta\lambda > 0$ there is one dS vacuum and one AdS vacuum. There will also be Bañados-Teitelboim-Zanelli black holes [15] as these are locally isometric to AdS, and it would be interesting to see how their microscopic degrees of freedom are encoded in some holographically dual $D = 2$ field theory.

In the context of $\mathcal{N} = 1$ supergravity, a solution is supersymmetric if it admits a nonzero spinor field ϵ satisfying $(D_\mu + \frac{i}{2} S\gamma_\mu)\epsilon = 0$, where S is the auxiliary scalar, constant in a vacuum. The integrability condition is $G_{\mu\nu} = -S^2 g_{\mu\nu}$, so $S^2 = -\Lambda$ in a supersymmetric vacuum. This condition is satisfied when $\beta = 0$ and $\lambda/\alpha \geq 0$, with $S = m\sqrt{\lambda/\alpha}$. For $\beta \neq 0$ the S field equation will be modified. Unfortunately, the modification depends on unknown coefficients of $S^2 R$ and S^4 terms in the action, so the status of AdS vacua of cosmological super-GMG remains an interesting open question.

Finally, in view of the ultraviolet finiteness of 4D gauge theories with $\mathcal{N} = 4$ supersymmetry, it seems likely that some supersymmetric extension of the new massive 3D gravity presented here will be not just renormalizable but ultraviolet finite.

PKT thanks the EPSRC for financial support, and the University of Groningen for hospitality. This work is part of the research programme of the *Stichting voor Fundamenteel Onderzoek der Materie* (FOM).

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